Exercise 46

If $h(2) = 4$ and $h'(2) = -3$, find

$$\frac{d}{dx}\left(\frac{h(x)}{x}\right)\bigg|_{x=2}$$

Solution

Evaluate the derivative using the quotient rule.

$$\begin{aligned}\frac{d}{dx}\left(\frac{h(x)}{x}\right) &= \frac{h'(x)x - \left[\frac{d}{dx}(x)\right]h(x)}{x^2} \\ &= \frac{h'(x)x - (1)h(x)}{x^2} \\ &= \frac{h'(x)x - h(x)}{x^2}\end{aligned}$$

Now evaluate it at $x = 2$.

$$\frac{d}{dx}\left(\frac{h(x)}{x}\right)\bigg|_{x=2} = \frac{h'(2)(2) - h(2)}{(2)^2} = \frac{(-3)(2) - (4)}{(2)^2} = -\frac{5}{2}$$